8. **Doppler effect :-** An apparent change in frequency of recieved radiation due to relative motion between source observer and medium is Known as Doppler effect.

Let us consider a plane wave where wave normal points in direction given by direction cosine l, m, n with respect to an observer in the frame S. Let its phase velocity be u such that its components are ul, um, un and let its wave length frequency and time period be λ, ν and T respectively such that.

$$\lambda = \frac{u}{\nu} = uT$$

These quantities in frame S' are written with primed symbols. Let us take an eqn a plane wave as

$$\psi = A\exp\left[2\pi i\left(\frac{lx+my+nz}{\lambda} - \frac{t}{\tau}\right)\right] \quad \text{—— (1)}$$

$$= A\exp\left[\vec{K}.\vec{x} - \frac{2\pi i t}{\tau}\right] \quad \text{—— (2)}$$

where vector $\vec{K} = 2\pi/\lambda\,(l, m, n)$ and points in the direction of wave propagation and $\vec{x} = (x, y, z)$

Now let us inductor a set of four quantities

$$(K_i) = [\vec{K}, i\omega/c]$$

where $\omega = 2\pi/\tau = 2\pi\nu$ with this introduction eqn (1) becomes

$\psi = A\exp(K_i, x_i)$ —— (3)

where (x_i) is a four position vector as defined subsequently in four vector. The inner product of (K_i) with four vector (x_i) gives a scaler quantities hence (K_i) is also a four vector with following usual spatial and temporal transformation as given in derivation of lorentz transformation relation.

$$\therefore \vec{K'} = \vec{K} + \frac{(\vec{K}.\vec{V})\vec{V}}{V^2}(\beta-1) - \beta\vec{V}\,\omega/c^2 \quad \text{—— (4)}$$

$$\text{and } \omega' = \beta[\omega - (\vec{K}.\vec{V})] \quad \text{—— (5)}$$

$$\text{where } \beta = \frac{1}{\sqrt{1-v^2/c^2}}$$

eq^n (1), (3) and (4) gives

the direction of propagation in frame S and S' while eq^n (5) inter relates the frequency in S and S'. Since we have

$$\vec{K} = 2\pi \vec{}/\lambda \cdot \vec{u}/u^2 = 2\pi\nu \, \vec{u}/u^2$$

Therefore eq^n (5) gives

$$\omega' = 2\pi\nu' = \beta\left[2\pi\nu - 2\pi\nu\left(\frac{\vec{u}\cdot\vec{v}}{u^2}\right)\right]$$

or $\nu' = \beta\nu\left[1 - \vec{u}\cdot\vec{v}/u^2\right]$ —— (6)

• This gives the relativistic Doppler effect and is different from classical case due to the factor

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}$$

If we apply it to the

case of light propagation in free space we have $u = c$ and for the source system attached to the frame S'.

We have $\nu' = \nu_0$.

Hence eqn ⑥ gives

$$\nu_0 = \nu\beta\left[1 - \vec{c}\cdot\vec{v}/c^2\right]$$

or $$\nu = \frac{\nu_0\sqrt{1 - v^2/c^2}}{\left(1 - \frac{\vec{c}\cdot\vec{v}}{c^2}\right)} \quad \text{——— ⑦}$$

In non relativistic case where v^2/c^2 we get

$$\nu = \frac{\nu_0}{\left(1 - \vec{c}\cdot\vec{v}/c^2\right)}$$